Virtual Dental Treatment Training System using a Haptic Device

햅틱장치를 이용한 치과 가상치료 시스템

Master Thesis

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WE HEREBY RECOMMEND THAT THE THESIS BY

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Virtual Dental Treatment Training System using a Haptic Device

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF **MASTER OF SCIENCE**

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This thesis is dedicated to my parents who
have given me so much, thanks for your faith in me.
And also dedicated to professor Kang Park who makes me write
such a document.

DECLARATION

I hereby declare that this work was done independently and without the use of other sources and tools than specified.

Furqan Ullah,
January 05, 2009

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Begins with the name of Almighty ALLAH, the creator of the entire universe, Who is the most merciful, beneficent, kind and generous.

Next to all His Messenger Hazrat Muhammad (Peace Be Upon Him) Who is an eternal torch of guidance and knowledge for humanity.

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ABSTRACT

Virtual Dental Treatment Training System using a Haptic Device

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This thesis presents virtual dental treatment training system with haptic environment in which dental students can learn dental procedures and experience of using dental cutting tool with realistic sense of touch. Surface based model for sculpting process simulation for less memory cost and better visual quality is utilized. In this thesis, data reduction algorithm for huge triangular mesh data is proposed in which we can separate one tooth from human jaw for getting higher visual update frequency (~30Hz) and stable haptic update rate (~1 kHz). Efficient collision detection can be realized with stable haptic force feedback interaction approach. For the computation of repulsive force during real time sculpting process, spring damper force model is used and a force filter is utilized to make repulsive force feedback smooth. Vertex deformation method is used for precision sculpting simulation with the integration of enhanced tri-subdivision of triangles algorithm. In order to consider fidelity, stability, computer efficiency and update rate of haptic simulation, this system can provide stable material removal simulation from human tooth with realistic sense of repulsive force.

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LIST OF SYMBOLS

SI, S2, S3	Side lengths of triangle
s	Semiperimeter of triangle
R_c	Circumcircle of triangle
R_i	Incircle of triangle
A, B, C	Vertices of triangle
x, y, z	Dimensions in world coordinate system
r	Radius of sphere
$\vec{N}$	Normal Vector
θ	Rotation angle of cutting planes
$\vec{P}_i$	Position of vertices
$\vec{n}_1$	Normal vector of fixed plane
$\vec{n}_2$	Normal vector of rotatable plane
V_j	ID number of ith vertex
$v1, v2, v3$	Vertices of triangle
$\overrightarrow{P0}$	Centroid of the 3D model
$\overrightarrow{P1}$	Corner position of cutting planes along “Z” axis
$\overrightarrow{P2}$	World coordinates at the desired position by mouse click
$\overrightarrow{P2'}$	World coordinates of rotatable plane at the desired position
$\vec{T}^c$	Position of center of haptic tool tip (sphere)
R_T	Radius of sphere
L	Magnitude of vector between vertex position and center of sphere
$\vec{N}_i^v$	Vertex normal vector of the ith vertex before updating
m	Total number of vertices of the tooth model
${}^u\vec{P}_i$	Updating position of ith vertex in 3D world coordinate system
${}^u\vec{N}^v{}_i$	Updating vertex normal vector of ith vertex

$\vec{F}_s$	Force exerted by the spring
k	Spring stiffness/constant
$\Delta\vec{d}$	Change in displacement from original spring position
$\vec{F}^t$	Total force vector
b	Viscosity
$\vec{v}_i$	Velocity of haptic tool tip
$\vec{F}_R$	Overall resultant force vector
$\vec{n}_i$	Vertex normal vector
$\vec{F}_f$	Final force vector
δ	Predefined threshold for the force change
U	User defined criteria

LIST OF FIGURES

Chapter 1: Motivation

The main objective of this research is to develop a software package for dental students in which they can perform cutting simulation for cavity preparation and removing of other tooth tissues in Virtual Reality environment with the realistic sense of touch. The dental treatment training is necessary for dental students in reality, because they need some experience before proceeding to the real patient and now with the help of this software they can perform cutting and learn same training as they do in real life. The graphical user interface (GUI) of virtual dental treatment training system is demonstrated in Fig. 1.1. To achieve this objective, real time 3D mesh reconstruction is utilized to transform a real human jaw model into the computer from 3D digital scanner data. A haptic device (SensAble) is used for the realistic sense of feelings. This research is very advantageous for dental students to perform sculpting on tooth in reality.

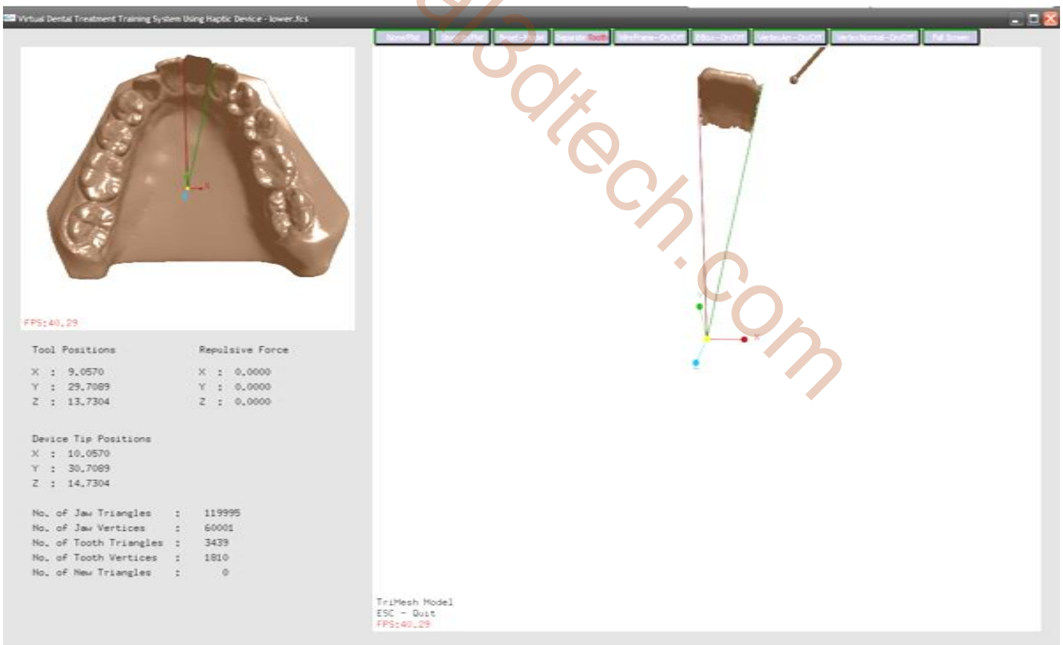


Figure 1.1: GUI of virtual dental treatment training system

Chapter 2: Introduction

2.1 Introduction of 3D:

The term 3D, or three-dimensional, means that an object being described or displayed has three dimensions of measurement: width, height, and depth. According to computer graphics point of view when we add “Z depth” in 2D “X, Y” object then we can say as a 3D dimensional object. For example a Rubik Cube has some width, depth and height as shown in Fig. 2.1 (a). And a piece of paper on a table would be a good example of two dimensional object due to have no perceptible depth. Fig. 2.1 (b) illustrated a piece of paper has no Z depth.

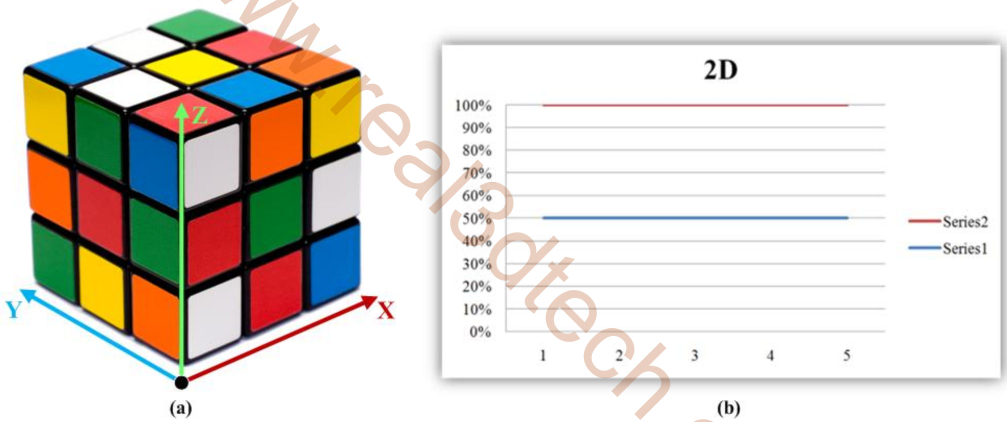


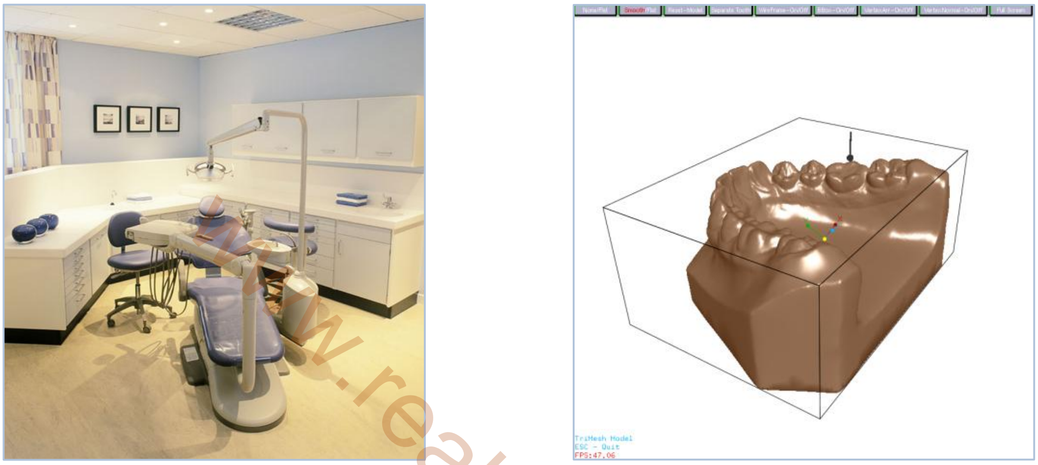
Figure 2.1: (a) 3D dimensional object “Rubik Cube”. (b) 2D object “Piece of Paper”

2.2 Introduction to VR Environment:

Virtual reality (VR) is a technology which allows a user to interact with a computer simulated environment. It is the simulation of a real or imagined environment that can be experienced visually in the three dimensions of width, height, and depth and that may additionally provide an interactive experience visually in full real-time motion with sound and possibly with tactile and other forms of feedback. Furthermore VR is the combination of two words, “Virtual” and “Reality” which means,

- If you can touch it and you can see it, it's REAL
- If you can't touch it but you can see it, it's VIRTUAL

An example of virtual reality can be illustrated in Fig. 2.2. More sophisticated effort involves in haptic device that let you feel the display images.



(a) (b)

Figure 2.2: (a) Dental treatment setup in real environment. (b) Human jaw in virtual environment

2.3 Basic 3D Structures:

All geometric primitives are described in terms of their coordinates that define the points, lines and polygons. Point is also called as vertex which represented in (x, y, z) dimensions in 3D environment. Line is also known as line segments and the representation of polygon is the area enclosed by single closed loops of line segments. Fig. 2.3 illustrated the basic structure of 3D objects lines and polygons.

Using an OpenGL library, beautiful 3D graphics can be presented with these basic geometric structures in exceptional visual quality. OpenGL is referred to as an API (application programming interface) in which a software interface to graphics hardware. It is used for various purposes, from computer aided design (CAD) engineering and architectural applications to modeling 3D programs. In this software application, OpenGL is used as a main graphics library to represent 3D model in virtual environment with triangle mesh.

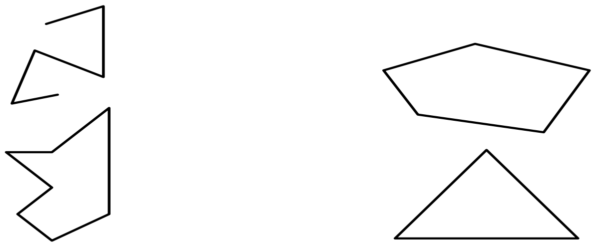


Figure 2.3: Basic structures of 3D geometry: (a) Connected series of line segments. (b) Polygons

2.4 Basic 3D Math and OpenGL:

For the computation of some algorithms and the representation of 3D model in this dental treatment training system, there is need to know about the basics of triangle calculations such as area in 3D, semiperimeter, circumcircle radius, and incircle radius of triangle which can be demonstrated in Fig. 2.4.

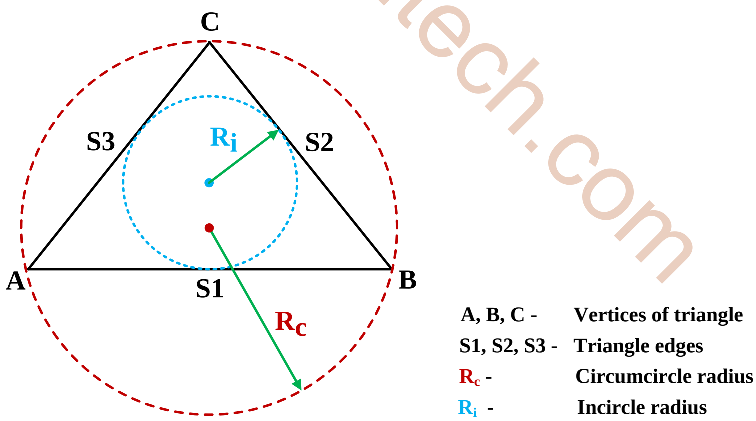


Figure 2.4: Basic triangle calculations

Eqn. 2.1 represents that the area of triangle in 3D with sides *S1* and *S3* is equal to the magnitude of the cross product of vector representing two adjacent sides.

$$Area\ of\ Triangle = \left|\overrightarrow{AC} \times \overrightarrow{AB}\right| \tag{2.1}$$

The circumcircle (R_c) can be compute by Eqn. 2.2. Where *S1*, *S2* and *S3* are the side lengths, s is the semiperimeter of triangle.

$$R_c = \frac{S1\ .\ S2\ .\ S3}{4 \times \sqrt{s(S1+S2-s)(S1+S3-s)(S2+S3-s)}} \tag{2.2}$$

Eqn. 2.3 shows that the incircle radius (R_i) can be calculated with the area of triangle divided by semiperimeter.

$$R_i = {}^{Area\ of\ Triangle}/_{s} \tag{2.3}$$

Where semiperimeter (s) is equal to the square root of the sum of all triangle edges.

$$s = \sqrt{S1 + S2 + S3} \tag{2.4}$$

By using Eqns. (2.5, 2.6, 2.7) the side lengths *S1*, *S2* and *S3* of the triangle can be computed.

$$S1 = \sqrt{(Bx - Ax)^2 + (By - Ay)^2 + (Bz - Az)^2} \tag{2.5}$$

$$S2 = \sqrt{(Cx - Bx)^2 + (Cy - By)^2 + (Cz - Bz)^2} \tag{2.6}$$

$$S3 = \sqrt{(Ax - Cx)^2 + (Ay - Cy)^2 + (Az - Cz)^2} \tag{2.7}$$

However, 3D points, triangles and models often need to be transformed in advance, before and after they are displayed on the screen. In virtual dental treatment system, translation and rotation of virtual dental round cutter during cutting process is required. Therefore, four kind of basic transformations are required: rotation, translation, scaling and skewing as demonstrated in Fig. 2.5. They don't describe anything in specific, but serve only to show matrix structure. To apply these transformations efficiently on many 3D objects, usually a 4x4 homogenous transformation matrix is employed.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation About X Axis **Rotation About Y Axis** **Rotation About Z Axis**

$$\begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling Matrix **Translation Matrix** **Identity Matrix**

Figure 2.5: Three dimensional common matrices

until now some basics about triangle have discussed because in this dental treatment training system, tooth model is actually constructed with thousands of triangles, so now need to introduce virtual dental tool which is constructed with polygon based sphere that can be illustrated in Fig. 2.6 with implicit form of equation.

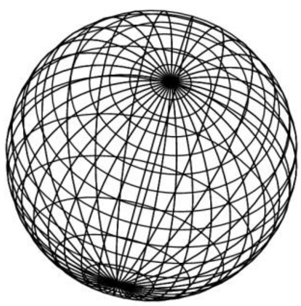


Figure 2.6: Sphere: $x^2 + y^2 + z^2 = r^2$

2.5 SensAble Haptic Device:

Simulations that include the feeling of touch provide a more realistic experience and better learning. The haptic interface used in this simulation is the PHANTOM Omni of SensAble Technologies; the device offers 6 degrees of freedom output capabilities. Also the PHANTOM haptic devices and OpenHaptics toolkit are used for academic and commercial research to create a wide variety of haptially-enabled applications. Fig. 2.7 illustrated a PHANTOM Omini haptic device which is used for the touch and manipulation of virtual 3D objects.

Figure 2.7: Haptic Device PHANTOM Omni: Haptic device is a mechanical device that mediates communication between the user and the computer. It makes it possible for users to touch and manipulate virtual objects in virtual environment. We can feel the realistic sense of touch during the cutting simulation of 3D tooth model in 3D dimensional virtual environment.

Chapter 3: Concept and Related Work

3.1 Introduction:

Simulation is an important feature in engineering systems or any system that involves many processes. Currently, in medical area simulators are applied to research and development of tools for new therapies, treatments and early diagnosis in medicine. Medical simulation such as dental treatment systems and surgery training systems etc. is area of major applications in computer graphics and virtual reality. A number of methods were presented for producing cuts in triangulated surfaces, these methods [5], [6], [11], [12] could be applied in surgery simulation. For the practice of dental operation artificial teeth and jaws models are available nowadays. These models cannot provide the level of detail and material properties of real-life teeth and procedures [2]. During cutting operations, haptic sensation is important for the dentists to implement the cutting successfully but the material property of artificial teeth is very different from that of original teeth so it is hard to get the real feel of sensation like real dental operation of a dentist. So consideration of repulsive force during material removal is important for the development of realistic dental treatment training system in virtual reality environment. Some work has been done in the haptic based cutting simulation [1], [4], [8], [9]. Novint is developing a VRDTS (Virtual Reality Dental Training System) prototype [13]. In VRDTS, students can perform drilling for cavity repair by using PHANTOM (Haptic device from SensAble). Mostly volume-based approach is used for cutting and drilling simulation since it provides direct and intuitive modeling without topological constraint unlike geometry-based one [9]. In recent years, many haptic force feedback simulators based on volumetric and surface data have been proposed for virtual simulation and visualization or medical training. Ranta et al. [1] introduced a concept design of a Phantom based dental training system to practice cavity preparation. Thomas et al. [4] developed a dental surgical simulator system; they also explain the software and implementation of the prototype system. However for realistic dental treatment simulation, they have many limitations such as feeling of repulsive force feedback during dental treatment, to get fast visually rendering, precise cutting simulation of dental tooth.

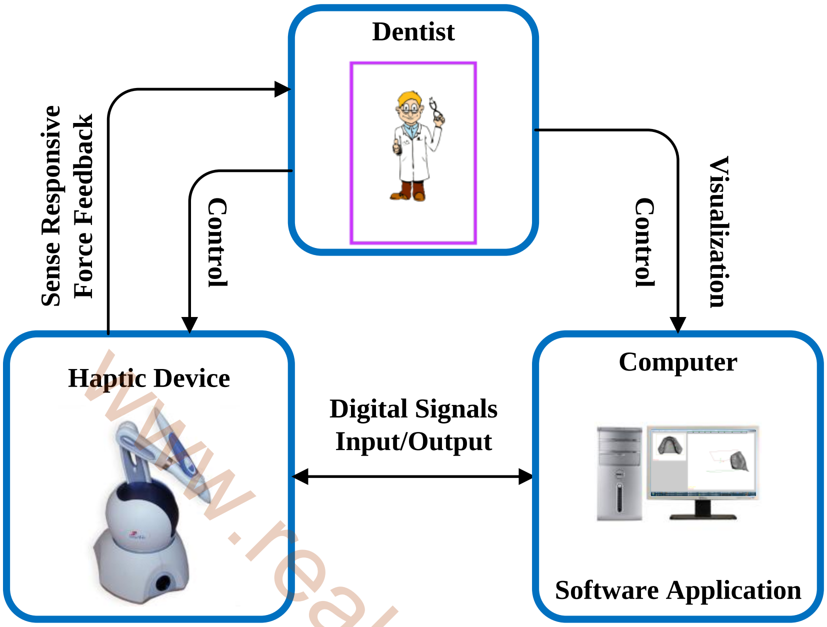


Figure 3.1: Basic concept of virtual dental treatment training system

In real life, dentists visualize the computer graphic and simulation, at the same time he also operate the virtual dental tool for the sculpting action. A haptic device interacts with software application by sending/receiving digital signals to computer. Then after every sculpting action, dentist sense realistic responsive force at haptic tool tip. Fig. 3.1. demonstrates the basic concept of whole procedure of dental training system with haptic display.

In this thesis, surface based virtual dental treatment training for dental students is proposed. Compared with the volumetric based approach, surface based approach is more complex and difficult to handle but costs lesser amount of computer memory. In real dental treatment simulation, tissues will be removed according to the interaction between the moving tool and tissues. This feature involves haptic sensation and computation of force rendering. For a realistic and smooth dental treatment, it’s very important to consider fidelity, stability and computer efficiency and also several important aspects such as collision detection, model regeneration and haptic force feedback. Based on these considerations, proposed surface based dental tooth model treatment training system has the following characteristics:

1. Data Reduction for Smooth Rendering: For smooth force rendering and cutting simulation, the tooth model data should be in small amount. But in surface based sculptures of human jaw have millions of triangles, so due to large amount of data, rendering of the model cannot be updated in real time due to small amount of memory as compared to model mesh data. First, it is needed to reduce the amount of data by efficiently separating of one tooth from human jaw. In this proposed system local-based data reduction algorithm is used in which not even one tooth but the desired tissue of tooth can be cut for more data reduction and for better understanding of students.

2. Enhanced Collision Detection: Efficient collision detection between cutting tool and sculpture tooth model is an important aspect for precisely material removal simulation. For realistic haptic sensation the dental training system must have ability to efficiently compute the responsive force feedback and realistic tool interaction with the virtual tooth, which all depends upon efficient collision detection. In this proposed method only one virtual tool is used for the collision detection and material removal from the desired part of tooth. The virtual tool has a bounding box (Bbox) that easily detects vertices of the tooth model that collide with the tool. This approach can reduce the time of rendering and get fast collision detection for a smooth haptic sensation.

3. Enhanced Haptic Sensation: The calculation of responsive force is depends on efficient interaction between virtual tool and surface of sculpture tooth model. For this purpose very famous Hooke's Law is used for the calculation of

repulsive force feedback. In Hooke's Law when force "F" is applied to spring, the spring will compress from one position to another and then the displacement can be computed with this compression. So spring force can be calculated by using this method. Even we compute the accurate responsive force; still we have a problem with smooth haptic sensation. For the handling of this problem, a smooth force signal is calculated.

4. Subdivision of Triangles: In surface based tooth model, subdivision of triangles during cutting process is an essential factor for smoothness of material removal edges after sculpting action. When the interaction between the virtual tool and the sculpture surface happens for material removal then the triangles size should be smaller than virtual tool diameter. In order to fulfill this condition a triangle subdivision algorithm should be applied to large triangles. And the triangle subdivision algorithm is also needed to eliminate the abnormality of some triangles that happens within the cutting area when the speed of cutting tool is fast and the operator moves the virtual tool continuously with the same speed. Joe Warren and Scott Schaefer [15] describe different kinds of subdivisions of surfaces i.e. Quad/triangle subdivision. Bruyns and Senger [6] describe surface cutting process with subdivision in large-scale simulations of various procedures. In this proposed system bi and tri-subdivision method for subdivision of triangles is used to efficiently handle abnormality of triangle problem.

5. Real Time Simulation in Virtual Environment: It is necessary to update the sculpting model during real time rendering after every single stroke of the cutting tool. After the deformation of vertices (sculpting process), the surface tooth model needs to be updated in real time to enable correct collision detection in the next computation cycle. But a typical system cannot update the physical model at a rate sufficient for haptic rendering (at least 1 KHz). In order to

reduce the update time for smooth haptic sensation, local updating of the surface model is performed.

Fig. 3.2 demonstrates the system architecture of virtual dental treatment training system which consists of collision detection, force computation, haptic rendering, subdivision and simulation processes.

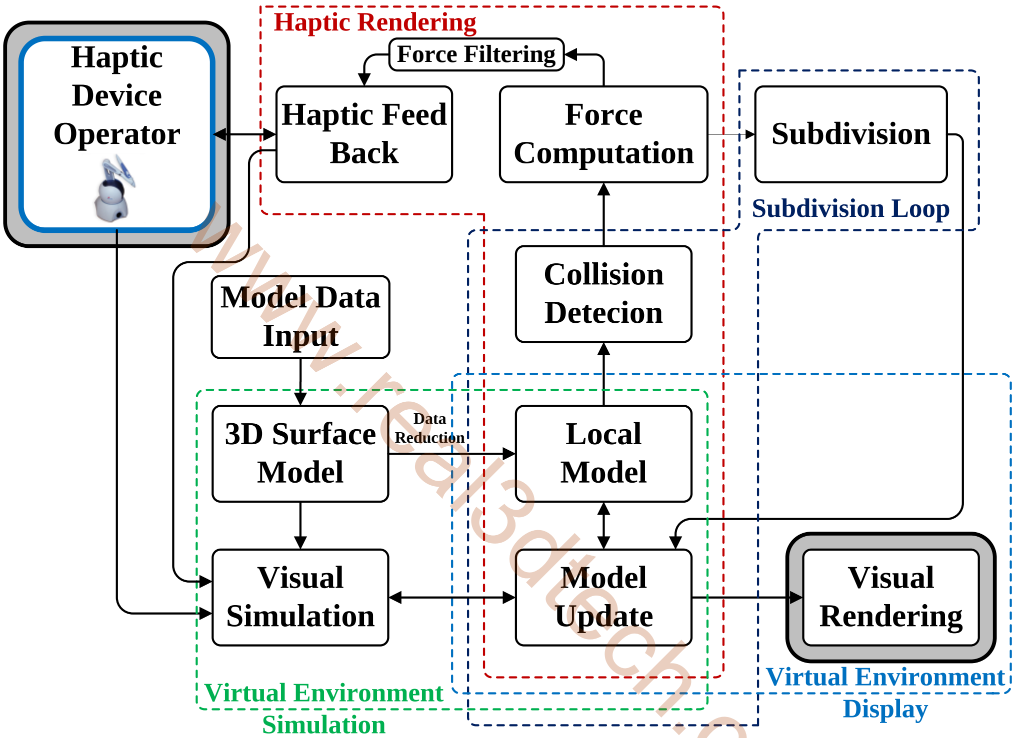


Figure 3.2: Virtual System Architecture

3.2 Related Work:

3.2.1 Virtual Reality Based Dental Treatment Simulation Systems:

During dental operation training, a trainee must learn to control surgical instruments very precisely, and develop a sophisticated feel for the interaction of these instruments with patient anatomy. Now days, most dental trainer learn fundamental procedures with plastic teeth or jaw, barely the excellent way to learn how to operate on live patients. Furthermore, with these dental treatment training methods, it is almost impracticable to measure a student's ability. To deal with these shortcomings, Novint [13] is developing a family of e-Touch Dental Simulators which integrate a very life-like feeling of touch, as well as 3D graphics and sound, to provide dental schools with realistic and quantifiable training systems. Practicing dental drilling/cutting for cavity preparation and other similar tasks is an important part of a dental student training. Usually students use artificial teeth and jaws and real instruments before proceeding to real patients. Recently, virtual reality based cavity preparation training systems have been introduced. The DentSim™ system (Denx Corp, Jerusalem, Israel) [18] comprises of a real dental unit, a manikin head, a tracking system and software that allows student to view the results of his/her cavity preparation in the manikin head on 3D models on a computer monitor and compare them with the results of most favorable preparation. Cutting on the clay using different shaped tools has been supported by the Freeform system [14]. H. T. Yau, L. S. Tsou and M. J. Tsai [19] have proposed Octree based virtual dental training system with haptic interface in which they propose a new approach that utilizes the so-called EP (Edge Proportion) value, local material stiffness and implicit function to develop a novel dental training system which has useful characteristics such as efficient collision detection, stable haptic interaction and accurate sculpting simulation. Hong-Tzong and Chien-Yu [20] present a dental training system which uses surfel (surface elements) models and carries out realistic cutting simulation using a haptic device. In this system a Boolean operation of the surfel model is implemented with local update; the modeling and visual update frequency of the system is about 30Hz.

3.2.2 Work of Voxel and Surfel based Dental Training Systems:

Voxel, a shortcut for VOlumatric piXEL, is the idea to represent every 3D object by a 3D grid containing thousands or sometimes millions of points. This grid can be represented just as one bit per voxel to save memory, or, if there is more memory available, color and transparency information can be stored as well. Pros and cons point of view it's very costly in memory consumption, 256*256*256 voxels equals 16 Mb data (one byte color depth). The most sophisticated algorithms (used to represent volumetric models) are Octree and Marching cube algorithms, multi-resolution and adaptive approach. In dental training system Octree is more well-known algorithm. In this case, every parent voxel consists of 8 child voxels. Barentzen [21] proposed an Octree based volume sculpting to reduce the memory requirement. However, these approaches have restriction and limitation like for example low resolution due to the data size of the volume, or the large amount of triangles for the displayed surface. H. T. Yau et al. [19] also presented octree-based virtual dental training system with a haptic device. Galyean and Hughes [22] introduced a voxel-based approach to volume sculpting that used the marching cube algorithm to display the model.

Surfel, a shortcut for SURface ELement [24]. Using surface based model costs lesser amount of memory than using volumetric representation. Hong-Tzong Yau and Chien-Yu Hsu, [20] used surfel model with Octree algorithm for the development and improvement of dental training system. Wang et al. [2] described the cutting on surface model by using haptic display in virtual reality environment.

There is big difference in representation of surface and volumetric rendering that is Voxel representation with volume rendering, Surface representation with standard rendering.

3.2.3 Haptic Interface in Virtual Reality Environment:

Many researchers are interested in using haptic feedback devices to enhance their virtual environment research because simulating touch in virtual worlds has led to increased performance in training simulators [1], [4], [7], [9], [10]. A haptic interface is a kinesthetic link between a human operator and the virtual environment in which computer generated model of some physically motivated scene [7]. The feeling of touch is important for medical training. Numerous diagnostic, surgical and interventional procedures involve that physicians train and use their sense of touch, which made useful and efficient medical training utilizing computers infeasible, until now. Novint's [13] technology and medical products add this missing component to computer-based training and simulation. Kim et al. [25] used a hybrid surface representation which is a combination of geometric model for virtual visual rendering and implicit surface for haptic display to take benefit of both data representations. Adams and Hannaford [7] discussed the basic stability and performance of haptic interface. Minsky et al. [26] explored stability problems in the haptic display of simple virtual reality environments. They noted a serious tradeoff between simulation update rate, virtual wall stiffness, and device viscosity and provided insights into the role of the human operator in stability concerns. A more accurate examination of the stability problem was performed by Colgate et al. [27]. They used a straightforward benchmark problem to obtain conditions under which a haptic display would show passive behavior.

3.2.4 Force Rendering:

During real time rendering of material removal, typically, visual frequency (~30 Hz) is much slower than haptic update rate (1 KHz). This performance gap leads is continuity in both the force direction and amount if the force is computed directly on the physical surface being sculpted. In order to smooth the force rendering, Foskey et al., uses spring-based force established between the initial contact point on the surface and tool tip position [28]. Hervé Delingette et al. proposed a tensor-mass force model similar to the spring-mass model [16]. A. Balijepalli and T. Kesavadas presented a force computation method of mechanical machining based on the principle of spring for the path planning of robot [29] [30].

3.2.5 Subdivision Approach:

Twenty years ago the publication of the papers by Catmull and Clark [31] and Doo and Sabin [32] marked the beginning of subdivision for surface modeling. Now we can regularly see subdivision used in movie production, smoothness of geometric models, cutting simulation in medical and in be a core technology in game engines. Recently subdivision surfaces have found their way into wide application in computer graphics and computer assisted geometric design (CAGD). One reason for this development is the importance of multi-resolution techniques to address the challenges of ever larger and more complex geometry. Constructing surfaces through subdivision sophisticatedly addresses numerous issues that computer graphics practitioners are confronted. Hui Zhang et al. introduce novel algorithms to subdivide the surface and generate interior structures for the simulation of the virtual cutting operation [33]. Kup-Sze Choi [34] described an interactive cutting simulation approach based on mass–spring system. He used subdivision of triangle algorithm during interactive cutting of deformable objects.

3.2.6 Collision detection:

The collision detection algorithm detects collisions between the 3D model and virtual dental tool and is also the foundation to compute force feedback for haptic rendering and update the tooth model. A. Gregory et al. presented a fast and accurate collision detection algorithm for haptic interaction with polygonal model [35]. The haptic toolkit developed by SensAble Technologies, also has a collision detection library which is useful for interaction of objects [36].

Chapter 4: Method

4.1 Data Representation:

4.1.1 3D Sculpture Model Representation:

In this algorithm, a local data structure is used to represent the 3D jaw model for dental treatment simulation because of its simplicity and efficiency. Mostly for sculpting, uses the uniform voxel data structure to represent an object, when precision increases, great quantity of voxel data will be produced to represent the object. As I mentioned above, using surface-based model costs less memory than using volumetric representation. Therefore, a local data structure is used to the representation of jaw model with less cost of memory and better fidelity. The original jaw model data is extracted from a digital 3D laser scanning data acquisition device which defines the boundary shape of the Jaw model in (*.fcs) file extension.

(a) **(b)**

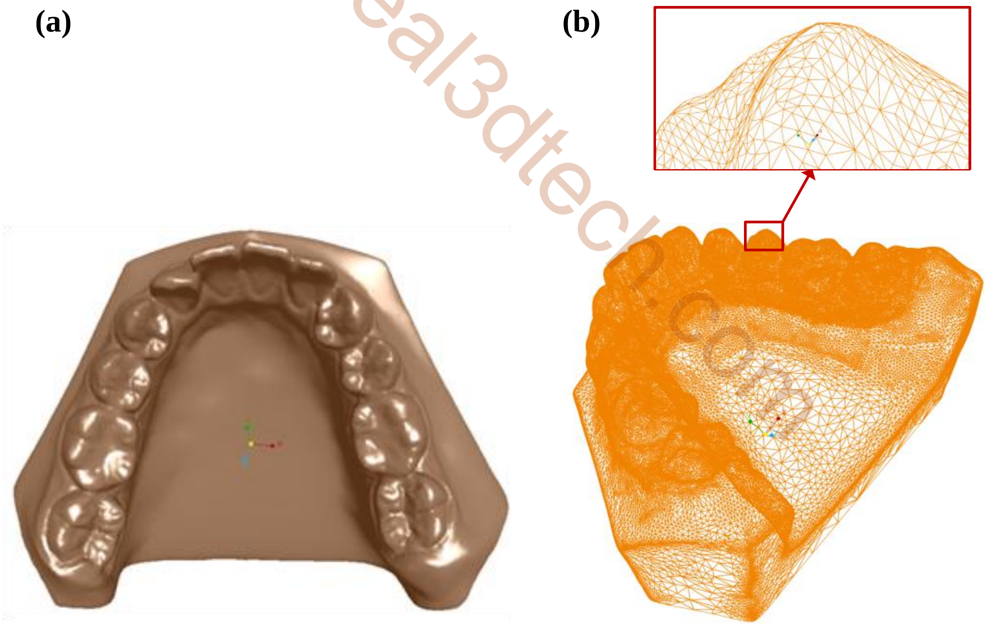


Figure 4.1 Model Representation, Scanned from Digital 3D Laser Scanner (a) Surface-Based 3D Jaw Model having 6001 vertices and 119995 triangles with Average Vertex Normal Vector. (b) Wireframe model of (a).

Fig. 4.1 (a) shows a surface-based 3D jaw model with smooth surface of enamel, dentin, pulp, cavity and several other triangles are represented as different kind of tissues of jaw. For switching solid model to wireframe model OpenGL gives a subroutine, by using built-in subroutine Figure 4.1 (b) shows back and front triangle mesh of jaw model. In flat representation of sculpture model each triangle has one normal vector for directing the light rays. But we need a smooth surface for better result during cutting process. So for this purpose a number of methods have been presented [37] in which they got smooth surface of 3D model by using different kind of subdivision of triangles i.e. Loop scheme, invented by Charles Loop [38] is shown in Fig. 4.2.

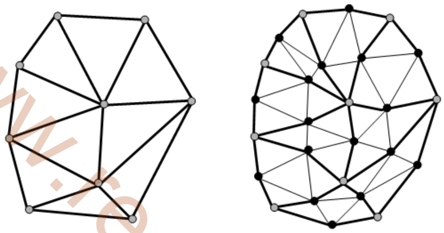


Figure 4.2: Loop Subdivision Scheme, Newly created vertices are shown as black dots. Every edge of the control mesh is divide into two, and new vertices are reconnected to form 4 new triangles, replacing every triangle of the mesh.

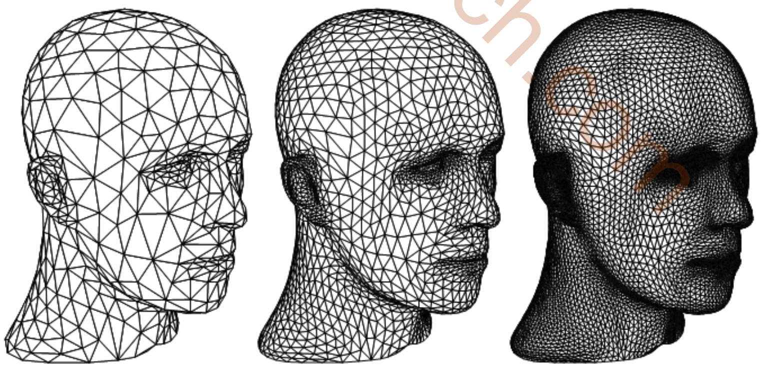


Figure 4.3: Case of subdivision for a surface, showing three successive levels of refinement. An initial triangular mesh on the left. Each triangle is divide into four according to a particular subdivision rule (middle). Mesh is subdivided once again (right).

Fig. 4.3 shows interpolating subdivision in which each triangle is subdivided into four triangles. So these subdivision methods for smooth surface will be very costly for proposed dental training system because of more memory usage.

Instead of using subdivision scheme Average Facet Normal method is used for smoothness of 3D sculpture model. In this method first we need to search all adjacent faces which share the vertex then the compute the average normal vector of all those adjacent faces to get the vertex normal as shown in Fig. 4.4.

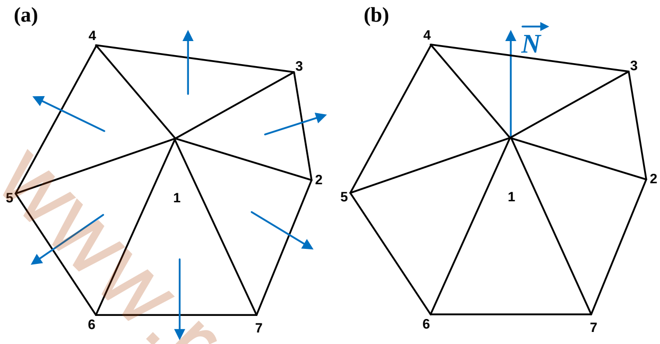


Figure 4.4: Facet Normal: (a) face normal vector of each triangle (b) The normals of six adjacent faces are averaged to compute the vertex normal in $\vec{N}$.

After computation of vertex normals then we can see a smooth surface of 3D model tissues. Fig. 4.5 illustrates the process of visualization of virtual 3D object in virtual environment from 3D point data file format.

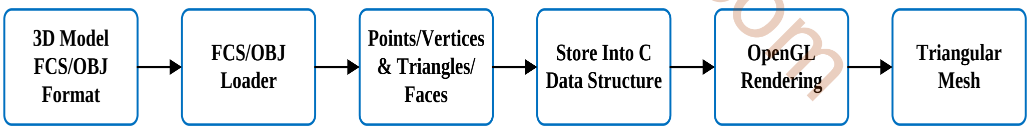


Figure 4.5: Rendering of virtual 3D object from FCS data file format.

4.1.2 Virtual Dental Tool Representation:

In real world, dentist use different kind of cutter shapes for sculpture the teeth according to the several requirements. Here a virtual dental round cutter tool is used which has a specified standard radius for cutting simulation as shown in Fig. 4.6. Dentist can change the size of round cutter tool any time during the real time cutting simulation. In this system two 3D objects are used in virtual environment, surface-based 3D tooth that constructed from triangle mesh and a virtual dental round cutter tool that constructed from a polygon based sphere. The tooth supposed to be still in world coordinate system but the dental tool can move around the 3D tooth freely along with the translation of haptic tool.

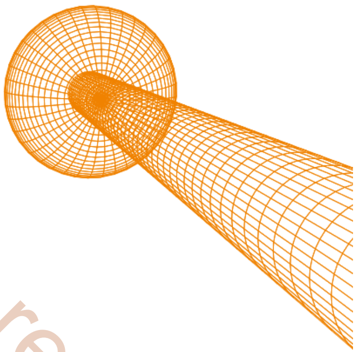


Figure 4.6: Virtual dental tool

4.2 Algorithms:

4.2.1 Data Reduction Algorithm:

During cutting process of tooth with haptic interference (i.e. cavity preparation etc.) higher update rate (or step size) is required in higher priority for smooth rendering of haptic display. But most of graphics card cannot render a huge data with millions of triangles very smoothly because model should be update before the next sculpting action. Due to this problem an algorithm is proposed in which we can reduce the original vertex or triangle data into small portion of model by creating two cutting planes. For example we can separate one tooth from the human jaw by creating two cutting planes which defines the boundary of separable tooth. Fig. 4.7 demonstrates the separation state of tooth from jaw model with two cutting planes.

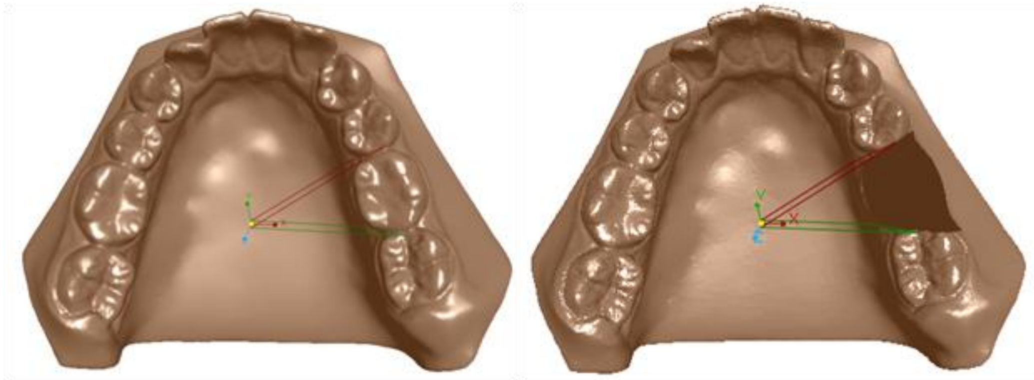


Figure 4.7: Data Reduction: (a) Surface-based 3D jaw model having 60001 vertices and 119995 triangles with two cutting planes. (b) After applying Data Reduction Algorithm, the black portion shows the removed tooth area.

The idea is when a dentist click by a mouse at any desired position in 3D space, two cutting planes will appear then set those planes at right position which define the boundary of the tooth as requirement and after apply data reduction algorithm, dentist will see a separate tooth from the human jaw. So for this purpose the following mathematical Eqn. 4.1 is used for the rotation of these cutting planes.

$$\begin{bmatrix} x^\circ{}_i \\ y^\circ{}_i \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{4.1}$$

Where x° and y° are the coordinates of rotatable cutting plane in 3D space, x and y are the coordinates of fixed cutting plane by mouse click which defines the right position of the planes, θ is the rotation angle of the rotatable cutting plane with fixed increment.
After computation of normal vectors of two planes, we need to check that which vertices of 3D model are inside between the two cutting planes then store into a new data stricture and outside vertices are discarded. Fig. 4.8 demonstrates the two cases when the angle (θ) between two planes is less than zero and greater than zero. Where $\vec{P_i}$ is the

vector position of vertices of triangles, $\vec{n}_1$ is the normals vector of rotatable cutting plane and $\vec{n}_2$ is the normal vector of fixed cutting plane.

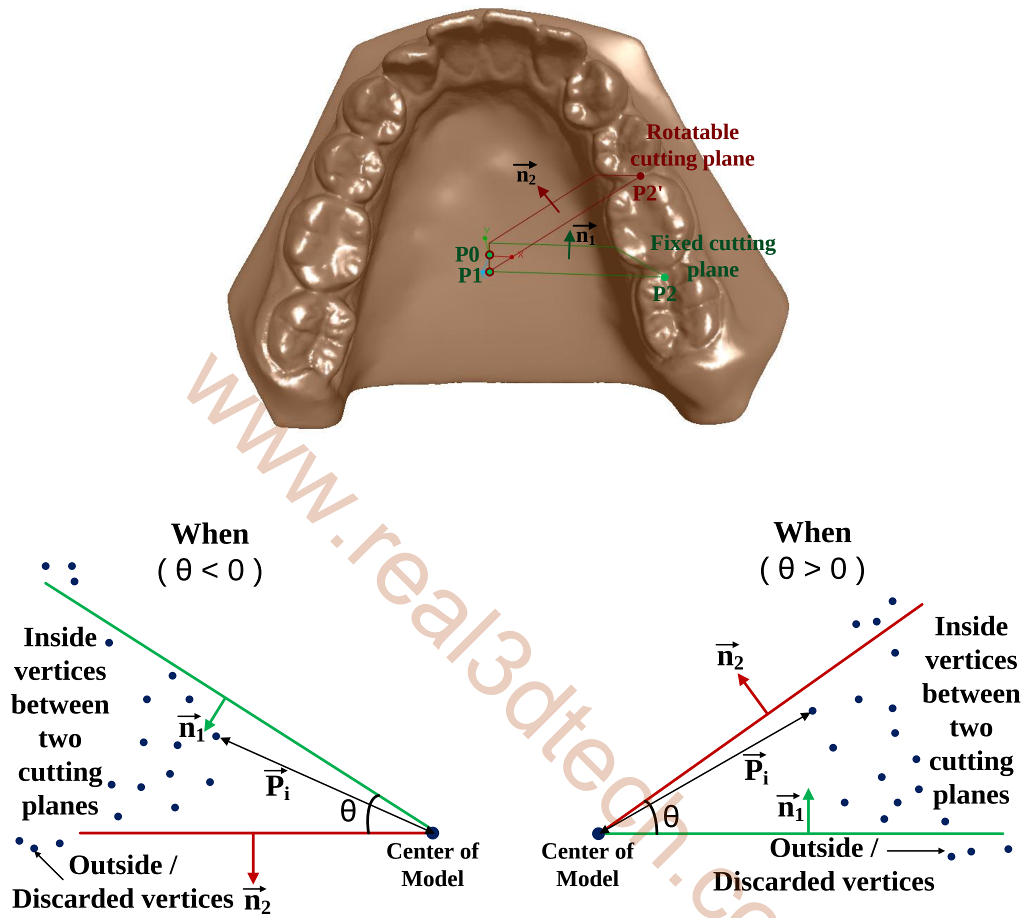


Figure 4.8: 3D cutting planes

So inside or outside vertices between two cutting planes can be checked by following algorithm which describes that if angle between two cutting planes greater than zero and if the DOT product of i^{th} vertex of the mesh and the normal vector of 1^{st} plane will be less than zero and if the DOT product of i^{th} vertex of mesh and the normal vector of 2^{nd} plane will be greater than zero then store all vertices ID information in V_j. Where V_j is the local data structure in which ID numbers of i^{th} vertex will be stored. When the angle between two cutting planes is less than zero then the case will be change that is, if angle between two cutting planes less than zero and if the DOT product of i^th^ vertex of the mesh and the normal vector of 1st plane is greater than zero and if the DOT product of i^th^ vertex of mesh and the normal vector of 2nd plane is less than zero then store all vertices ID information in V_j.

$$
\begin{aligned}
&if\ (\theta > 0\ ) \\
&\quad if\left(\ \vec{P}_i \cdot \vec{n}_1\ <\ 0\ \ \&\&\ \ \vec{P}_i \cdot \vec{n}_2\ > 0\ \right) \\
&\qquad V_j = \text{ID number of i}^{\text{th}}\text{ vertex} \\
\\
&if\ (\theta < 0\ ) \\
&\quad if(\ \vec{P}_i \cdot \vec{n}_1\ >\ 0\ \ \&\&\ \ \vec{P}_i \cdot \vec{n}_2\ < 0\ ) \\
&\qquad V_j = \text{ID number of i}^{\text{th}}\text{ vertex}
\end{aligned}
$$

$$
V = \begin{bmatrix} v1 \\ v2 \\ v3 \end{bmatrix} \quad \begin{matrix} 1st\ Vertex\ of\ Triangle \\ 2nd\ Vertex\ of\ Triangle \\ 3rd\ Vertex\ of\ Triangle \end{matrix}
$$

$$
\vec{n}_1 = \frac{(\ \overrightarrow{P1} - \overrightarrow{P2}\ ) \times (\ \overrightarrow{P0} - \overrightarrow{P2}\ )}{\|(\ \overrightarrow{P1} - \overrightarrow{P2}\ ) \times (\ \overrightarrow{P0} - \overrightarrow{P2}\ )\|} \tag{4.2}
$$

$$
\vec{n}_2 = \frac{(\ \overrightarrow{P1} - \overrightarrow{P2'}\ ) \times (\ \overrightarrow{P0} - \overrightarrow{P2}'\ )}{\|(\ \overrightarrow{P1} - \overrightarrow{P2'}\ ) \times (\ \overrightarrow{P0} - \overrightarrow{P2'}\ )\|} \tag{4.3}
$$

$\overrightarrow{P0}$ = Centroid of the 3D model

$\overrightarrow{P1}$ = Corner position of cutting planes along "Z" axis

$\overrightarrow{P2}$ = World coordinates at the desired position by mouse click

$\overrightarrow{P2'}$ = World coordinates of rotatable plane at the desired position

After that we assign new vertex ID numbers to all which are inside between the two cutting planes and store vertices position and IDs into two different local data structures. With the help of new data structures we rendered a separate tooth with very less amount of memory. This algorithm will be helpful in smooth translation of haptic tool tip and fast model updating during cutting process of tooth.

4.2.2 Collision Detection Algorithm with Cutting Process:

A realistic dental training system with haptic interaction requires natural and real time interaction between cutter and tooth surface. Here a sphere as a round cutter is used which have center $\vec{T}^c$ (T_x^c, T_y^c, T_z^c) corresponding to the position of center of haptic tool tip. The sphere has a bounding box which intersects the surface of tooth before the sphere surface and has the same center position as sphere center. Sphere's Bbox is used for the reduction of vertices data computation, because we need to check that how many vertices penetrate into the Bbox for the calculation of offset distance. Here it is also need to know about the area of triangle, circumcircle radius and incircle radius of triangle relevant to penetrated vertex for the regular or irregular behavior (abnormality of triangles) during continuous deformation of vertices. The intersection between tooth surface and virtual Bbox can be illustrated in Fig. 4.9, where R_T is the radius of sphere and $\vec{P}_i$ (P_x, P_y, P_z) is the position of penetrated vertex from the center of tooth model in 3D world coordinate system.

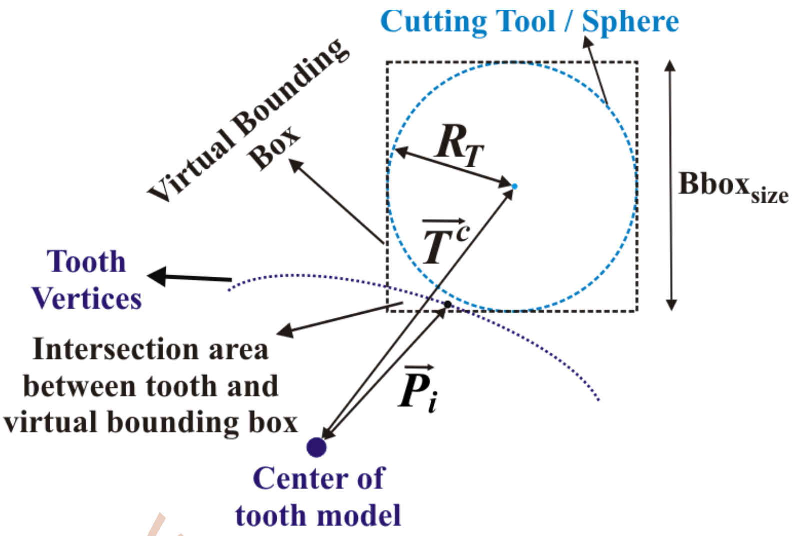


Figure 4.9: Intersection between virtual Bbox and tooth surface

The intersection between sphere's Bbox and tooth vertices can be computed by following condition check.

$$if\ (\vec{P}_i < \vec{T}^c + \frac{Bbox_{Size}}{2} \quad \&\& \quad \vec{P}_i > \vec{T}^c - \frac{Bbox_{Size}}{2})$$

Then *store* i^{th} *penetrated vertex into Bbox;*

Then Euclidian distance between vertex position in 3D space and the center of sphere can be calculated by following equation.

$$L = \left\|\vec{P}_i - \vec{T}^c\right\| \tag{4.4}$$

Where L is describing the magnitude of vector between vertex position and center of sphere. Material removal operation only occur when the virtual tool intersects with the tooth surface then we check, which vertex penetrate into the sphere, if the distance (L) of penetrated vertex from center of sphere is less than the radius (R_T) of sphere, then we consider that penetrated vertex for the deformation calculations, otherwise discarded.

Fig. 4.10 (a) demonstrates the collision between cutting tool/sphere and the surface of tooth model and Fig. 4.10 (b) shows the deformed vertex into the sphere surface after computation of updating vertex normal vector and the updating position of $\vec{P}_i$. Where $\vec{N}_i^v$ in Fig. 4.10 (a) is the vertex normal vector of the i^{th} vertex before updating which is opposite to the deformed direction and can be calculated by Eqn. 4.7. *m* is the total number of vertices of the tooth model.

$$\vec{N}_i^v = Vector\ Normalize \left[\sum_{i=0}^{m} \vec{N}_i \right] \tag{4.5}$$

In order to calculate the updating position of vertex which is at the sphere surface, so we need to extend that penetrated vertex with the vertex normal, after that model is updated and formed final cutting section of tooth illustrated in Fig. 4.11 (d), where $\Delta\vec{d}_i$ is the depth of cut of dental tool which means that the distance of deformed vertex from the original tooth mesh to the sphere surface. Finally, Eqn. 4.9 described the destination position (sphere's surface) of penetrated vertices.

$${}^{u}\vec{P}_i = \left(R_T \, . \vec{N}^v{}_i\right) + \vec{T}^c \qquad for\ i,\ L < R_T \tag{4.6}$$

$${}^{u}\vec{N}^v{}_i = \vec{P}_i - \vec{T}^c \Big/ L \tag{4.7}$$

Where R_T is the radius of sphere and ${}^{u}\vec{N}_i^v$ is the updating vertex normal vector of i^{th} vertex and ${}^{u}\vec{P}_i$ is the updating position of i^{th} vertex in 3D world coordinate system. In order to consider 1 kHz frequency for haptic rendering and 30 Hz for visual frequency, this collision detection algorithm gives smooth cutting simulation.

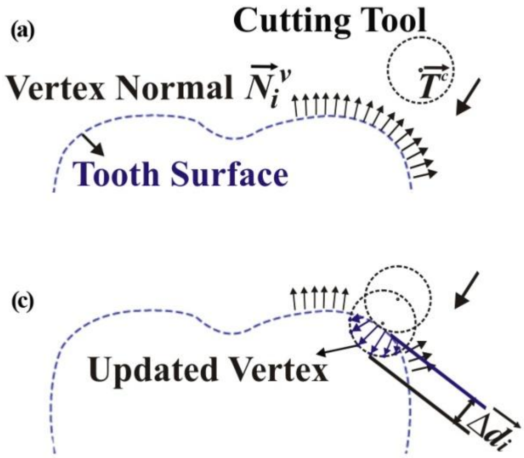


Figure 4.11 Collision Detection with Cutting Process: (a) No collision between virtual tool and tooth surface. (b) Intersection between virtual tool and tooth surface. (c) After intersection, the vertices deformed from original tooth surface to the virtual tool surface. (d) After cutting, final tooth shape

4.2.3 Haptic Rendering Algorithm:

Generally graphics applications in real time virtual environment have display update requirement almost between 20 to 30 frames per seconds. But for the sense of touch, the update rate of haptic simulations must be as high as 1 kHz frequency in order to maintain a stable force feedback system. Stiffness of displayed forces and the motion speed of the user may affect the variation of this haptic updating rate. In order to find the force vector, first need to find the intersection between tooth surface and haptic tool tip as demonstrated in Fig 4.10. As discussed in literature, the spring force model is commonly used for the transformation from the motion signal of the haptic tool tip to the virtual force signal. So the spring damping model for the computation of force feedback is used.

$$\vec{F_s} = k \, . \Delta\vec{d} \tag{4.8}$$

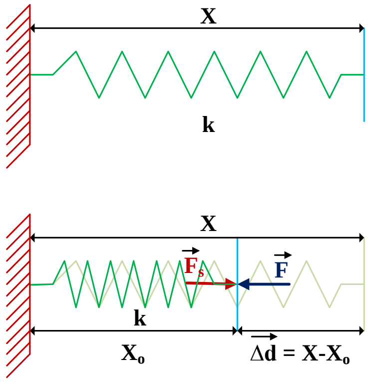


Figure 4.12: Hooke's Law for spring force computation

Where $\vec{F}_s$ is the force exerted by the spring, k is a spring stiffness/constant and $\Delta\vec{d}$ is the change in displacement from original spring position. After the intersection between tooth surface and sphere then need to know that how many vertices of tooth mesh penetrate into the sphere in one stroke of cutting tool because here need to know the total force of all penetrated vertices before sending to the haptic device. The direction of computed force vector is opposite as the direction of normal vector of deformed vertices illustrated in Fig. 4.13. This amount of computed force of one vertex is proportional to the distance between the sphere center and the sphere surface.

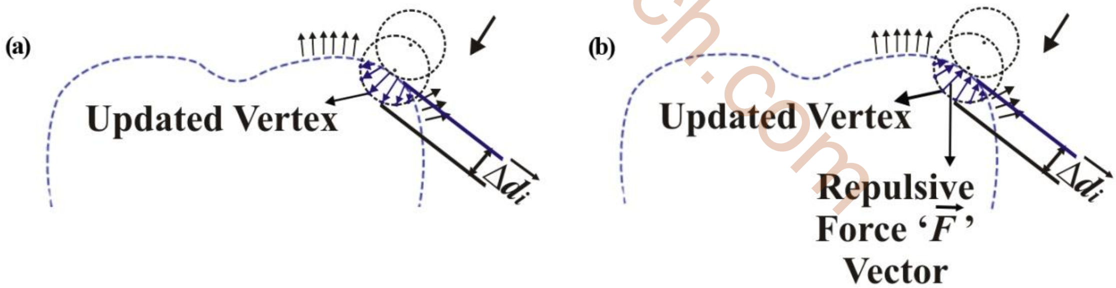


Figure 4.13: Force computation: (a) Deformation of vertices from original tooth mesh to the sphere surface. (b) Force exerted by deformed vertices

After summation of all forces of all penetrated vertices, the total force vector $\vec{F}^t$ (F_x^t, F_y^t, F_z^t) by apply the spring-damping model can be computed by Eqn. 4.11.

$$\vec{F}^t = k \ . \ \sum_{i=0}^{m} \Delta\vec{d}_i \tag{4.9}$$

Finally it must be necessary to consider material properties b and $\vec{v}_i$ for the total amount of 3D force. Eqn. 4.12 shows the overall resultant force before sending to the haptic device controller.

$$\vec{F}_R = k \ . \ \sum_{i=0}^{m} \Delta\vec{d}_i \ .\vec{n}_i - \ b \,.\vec{v}_i \tag{4.10}$$

Where b is the viscosity, $\vec{v}_i$ is the velocity of haptic tool tip, m is the number of penetrated vertices into the sphere tool, $\vec{n}_i$ is the vertex normal vector and $\vec{F}_R$ is the overall resultant force vector.

4.2.4 Force Filtering:

For smooth force feedback several different methods have been implemented in dental simulation systems. All those methods tried to develop a stable haptic interface system with 1 kHz frequency of haptic device. Under the consideration of haptic update rate 1 kHz, we use [19] Eqn. 4.13 to compute the final force vector and finally send to the haptic device controller.

$$\vec{F}_f = \vec{F}_{f-1} + \delta \,.\frac{\vec{F}_R}{\|\vec{F}_R\|} \qquad \|\vec{F}_R\| > \delta \tag{4.11}$$

Where δ is a predefined threshold for the force change. Fig. 4.14 illustrates the process algorithm for the final force computation.

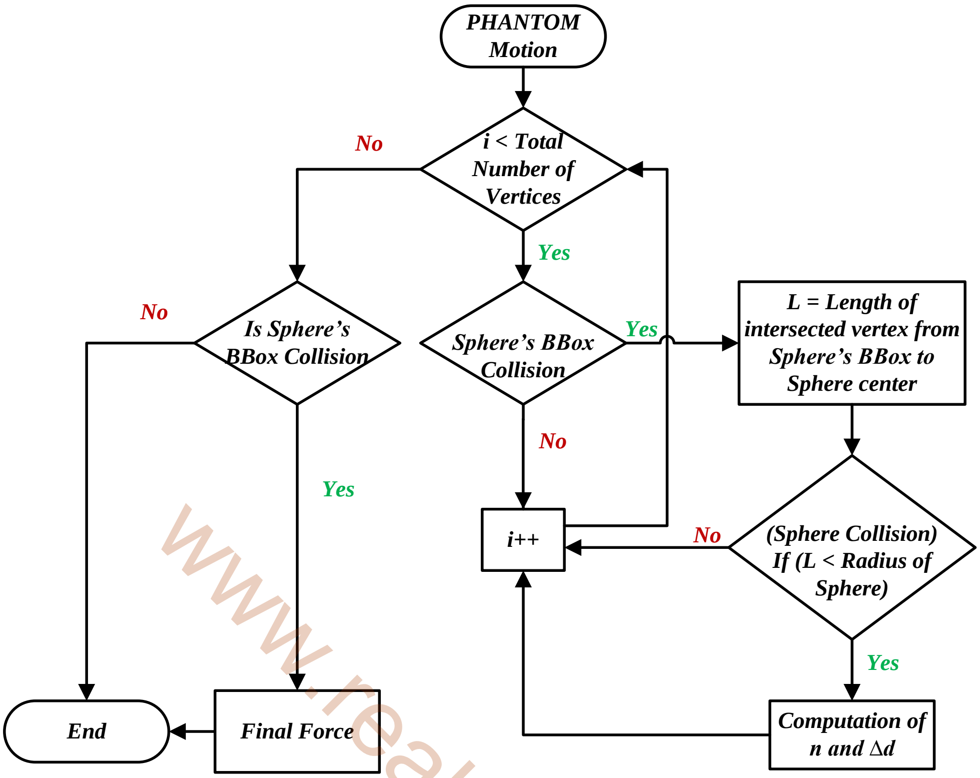


Figure 4.14: Force computation algorithm

4.2.5 Subdivision Algorithm:

As discussed previously that abnormality of triangle happens during continues deformation of tooth vertices by cutting tool, such as long and thin triangles which will affect the stability of force computation and tooth shape. In order to handle this problem reconstruction method [2] of triangles is used within the local cutting area. In which tri-subdivision method is used for the correction of abnormality of triangles. According to tri-subdivision method a midpoint of two longer edges will be selected for the division of triangle from one to three triangles. But at the same time the adjacent two triangles also need to be reconstructed for the stability of local mesh by bi-subdivision method in which one triangle will be converted into two triangles. In our 3D mesh model one triangle is connected with many triangles and makes different kind of combinations with different number of triangles. For the better understanding we use six triangles for the illustration of subdivision process in six possible cases. Fig. 4.15 illustrates the six possible cases of tri-subdivision and bi-subdivision method.

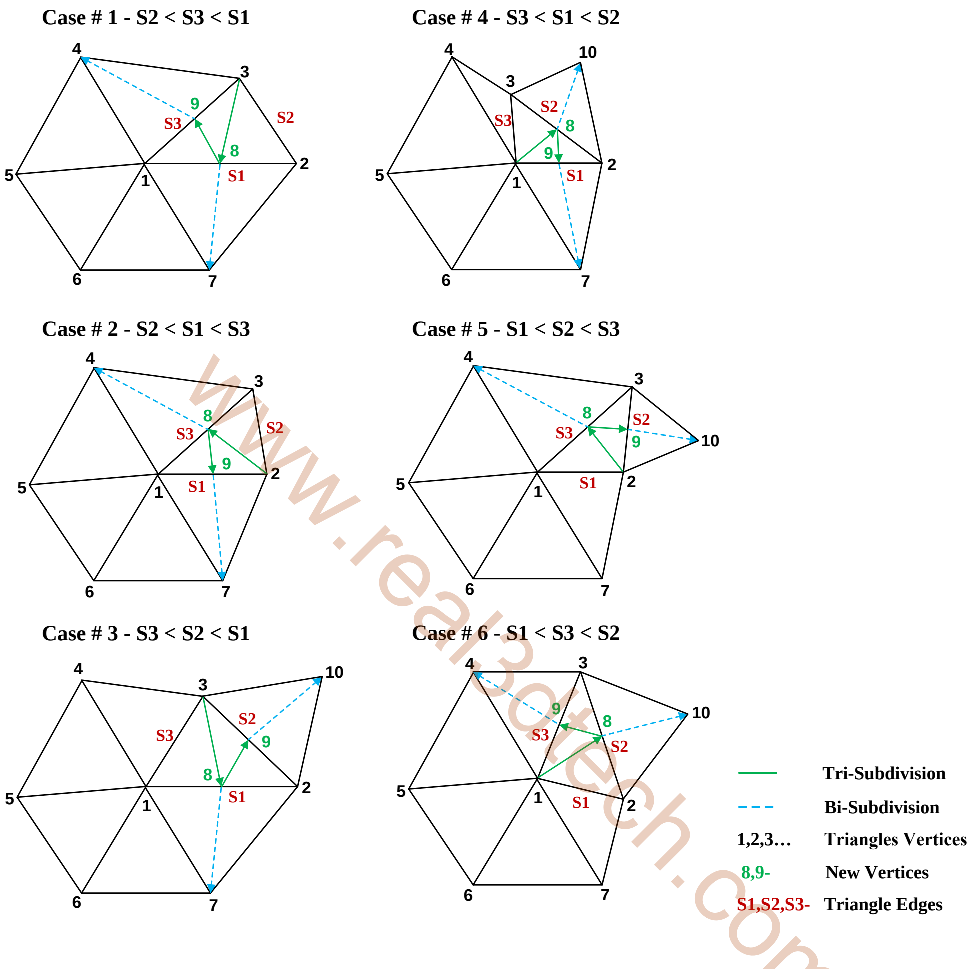


Figure 4.15: Tri-Division Method, 6 possible cases of subdivision of triangle

According to algorithm point of view after the deformation of vertices we need to know that which triangle lost its normality behavior within the local deformation area and then we make a check. For this purpose we computer some basic triangle information such as area, incircle radius, circumcircle radius of only local triangle are computed and normality of triangle is judged by a criteria with the ratio of incircle and circumcircle of local triangle by following equation.

$$if\ \left(\frac{Rc}{Ri} > U\ \&\&\ ((S1\ >\ L)\ OR\ (S2 > L)\ OR\ (S3 > L))\right)$$

***Then** active subdivision;*

Where *Rc* is the circumcircle radius, *Ri* is the incircle radius, *S1*, *S2*, *S3* are the magnitudes of edges of local triangle. *U* and *L* are the magnitude of user defined criteria for the activation of subdivision process.

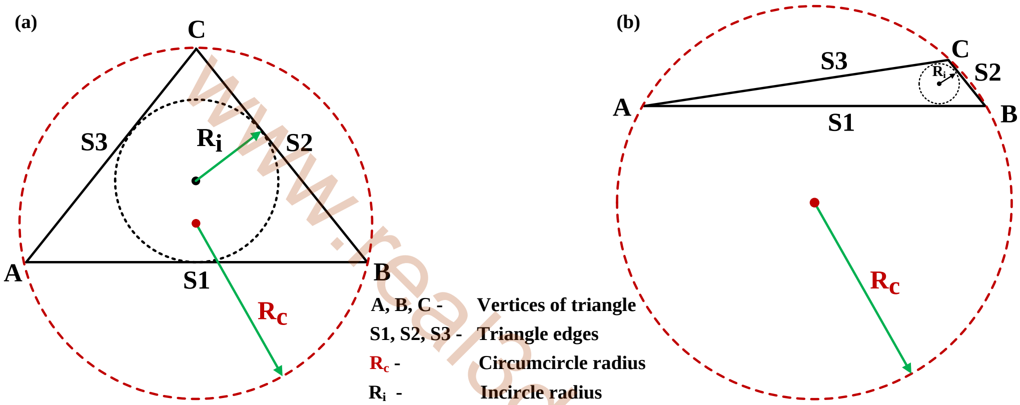


Figure 4.16: (a) Regular triangle. (b) Irregular triangle

After the reconstruction of mesh in real time cutting process, model needs to be updated due to change in vertices and the normal information. Because the updating with huge surfel mesh data is very costly for computer memory and real time rendering of haptic force feedback, so for the enhancement of cutting simulation, we use local updating method after subdivision.

4.2.6 Local Updating:

After every sculpting acting model should be update before the next sculpting action. So for fast sculpting and haptic rendering, data updating speed should be fast.

For this purpose we use local updating because the surface point data is not change very much during sculpting process. We only consider deformed area for updating and reconstructing. First, the collision occur between tooth surface and the haptic tool tip then compute force and update the information of force signals then send to the haptic device, Fig. 4.14 illustrated the force computation process. Then after the vertices and normals information in deformation process is updated as shown in Fig. 4.11. Subdivision of triangle process is considered at the end because in some cases subdivision doesn't happen, when the triangles behavior will be in normal condition during sculpting process, six cases of subdivision of triangle illustrated in Fig. 4.15. There are two kind of major data updates during start to end sculpting process, Fig. 4.17 illustrates the full process description of virtual dental treatment training system using haptic device.

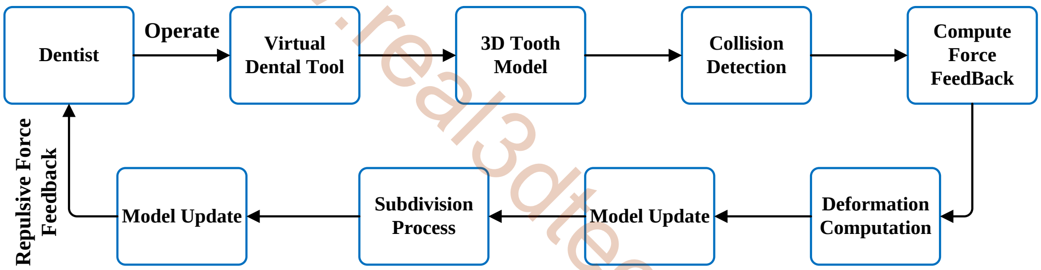


Figure 4.17: Process flow of virtual dental treatment training system

Chapter 5: Implementation and Results

The specification of the proposed dental treatment training system in virtual environment is 2.00 GHz Intel core™2 CPU, 2 Giga Byte of RAM and the simulation rendering based on Nvidia 8800GT GPU. The setup can be shown in Fig. 5.1.

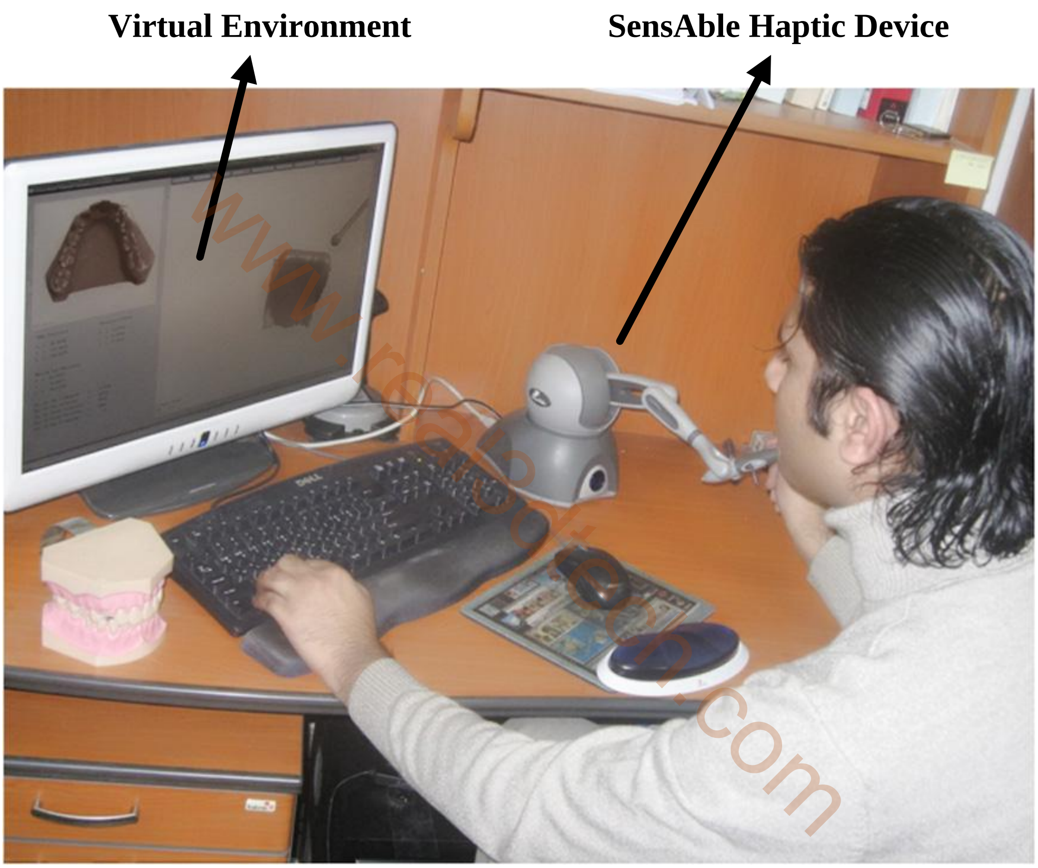


Figure 5.1: The Setup

5.1 Implementation of Data Reduction Algorithm:

First of all data reduction algorithm is implemented and result is demonstrated in Fig. 5.2-5.3.

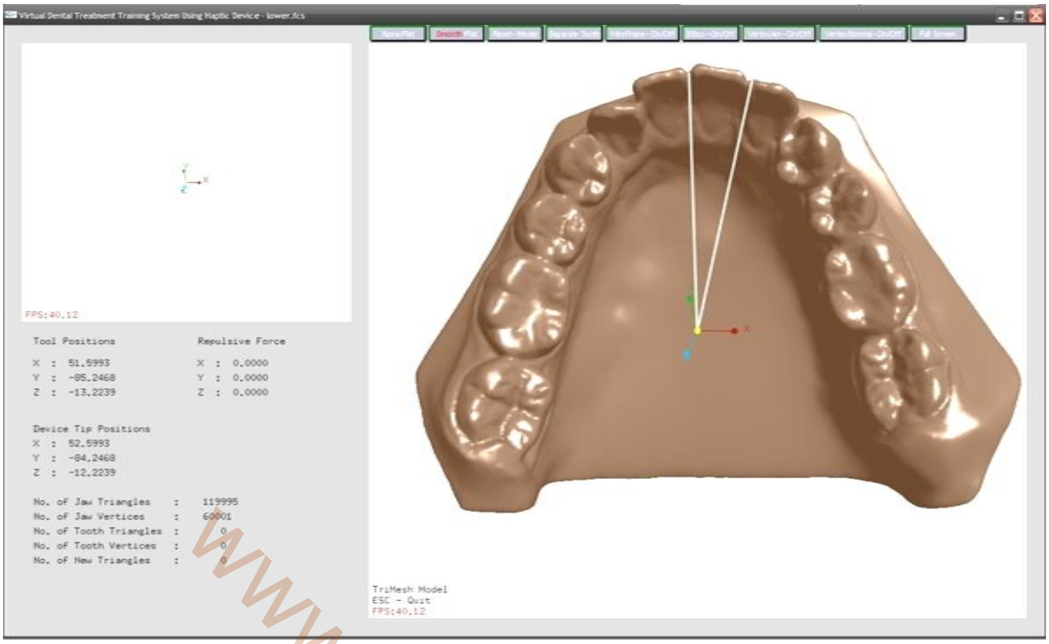


Figure 5.2: Before implementation of data reduction algorithm.

No. of triangles = 119995, No. of vertices = 60001

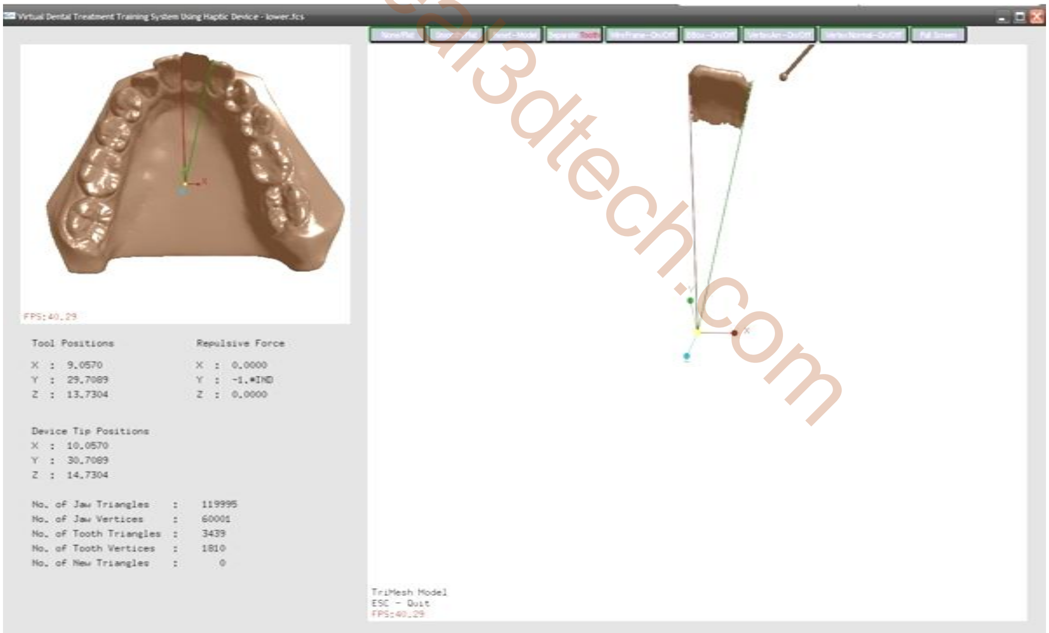


Figure 5.3: After Implementation of data reduction algorithm

No. of triangles = 3439, No. of vertices = 1810, Percentage of remaining data = 2.86%

5.2 Implementation of Sculpting:

The operation of real time cutting simulation is implemented which can be shown in Fig. 5.4.

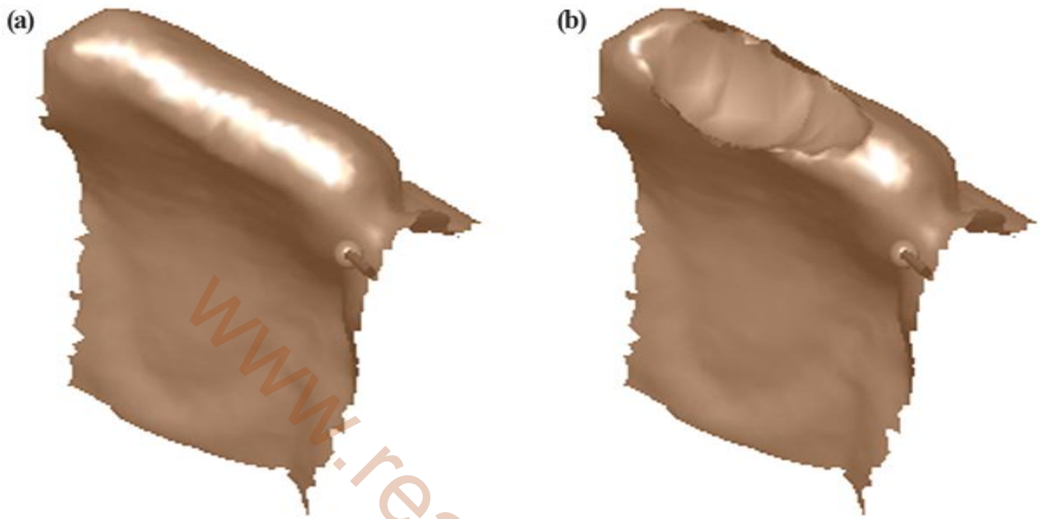


Figure 5.4: Material removed from tooth tissues by implementation of real time collision detection and deformation of vertices algorithm. (a) Before implementation of sculpting. (a) After sculpting

5.3 Implementation of Tri-Subdivision Algorithm:

Tooth triangle mesh is very complicated, so for the understating of subdivision of triangles process, a simple example is illustrated in Fig. 5.5.

Real time subdivision of triangles with tri-subdivision method during sculpting process can be illustrated in Fig. 5.6.

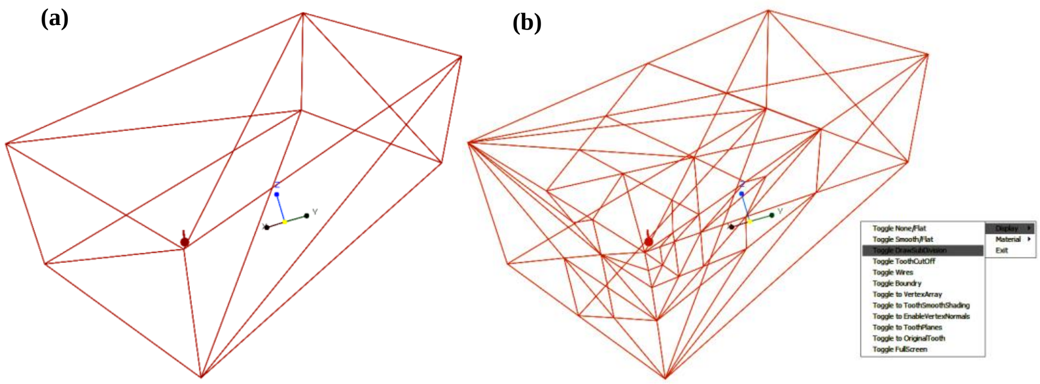


Figure 5.5: A simple rectangle mesh data exported from CAD software Pro-ENGINEER. (a) Before subdivision of triangles because there is no collision between rectangle and dental virtual dental tool. (b) After collision, the rectangle triangles are subdivided into many triangles.

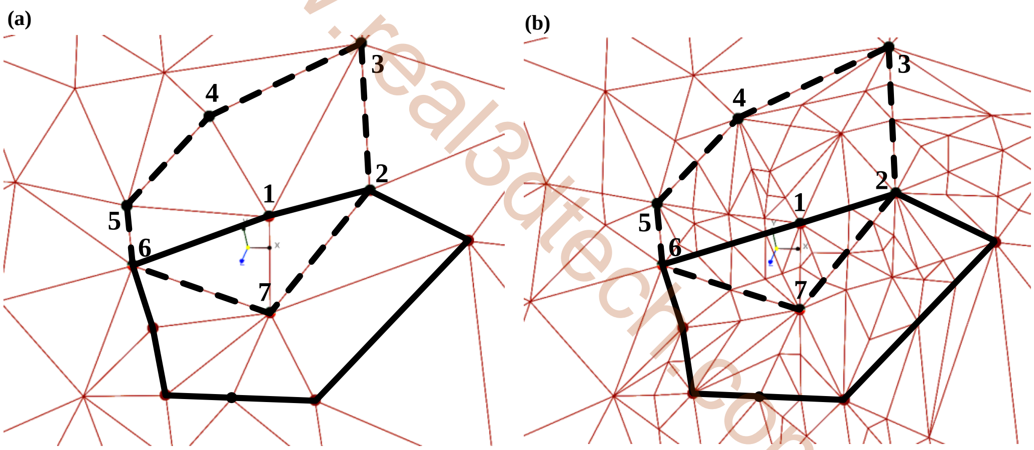


Fig. 5.6. (a) Original tooth model mesh before subdivision of triangles in real time sculpting simulation. (b) After subdivision of triangles during sculpting simulation.

Chapter 6: Conclusion and Future Work

In this thesis virtual dental treatment training system is presented in which dental students can learn dental procedures with realistic feelings of touch. Surface based virtual dental cutting simulation is performed using haptic interface. It is concluded that the mapping from haptic tool tip to the virtual tool is very critical for designing the force computation model. It is also concluded that using Surface based model costs less memory than using volumetric representation. Data reduction algorithm is proposed for the fast rendering of sculpting simulation. Vertex deformation method is used for the removal of tooth tissues, and for the prevention of abnormality of triangles, tri-subdivision method is utilized. Enhanced model updating by using local based updating algorithm. For future work, researchers need to include friction force for better sense of touch. Material removal from tooth surface needs to improve when continues deformation occur. Different visual and audio effects can be added for better understanding of dental students.

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